Topic: Ladder sliding down the wall

Question: At what rate is the shovel sliding along the ground?

A gardener's shovel is 1 m long and leaning against a fence. When the top of the shovel is sliding down the fence at a rate of 0.25 m/s. When the top of the shovel is 0.5 m off the ground, at what rate is the bottom of the shovel sliding along the ground away from the fence?

Answer choices:

A $\frac{3\sqrt{3}}{4}$ m/s

B $\frac{4\sqrt{3}}{3}$ m/s

C $\frac{4}{3}$ m/s

D $\frac{\sqrt{3}}{12}$ m/s

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Solution: D

The gardeners shovel, the fence and the ground form a right triangle in which the shovel is the hypotenuse. Since we know that the equation of a right triangle is given by $a^2 + b^2 = c^2$, we can say that a is the distance along the ground from the fence to the bottom of the shovel, that b is the distance along the fence from the ground to the top of the shovel, and that c is the hypotenuse of the triangle and the length of the shovel.

As with any related rates problem, the first thing we should do is write down what we know.

Change in b with respect to time $\quad \dfrac{db}{dt} = -\dfrac{1}{4}$

Note: We say that db/dt is negative because the top of the shovel is sliding down the fence, which means that length of the side b of our right triangle is getting smaller.

Length of the shovel $\quad c = 1$

Equation of the right triangle $\quad a^2 + b^2 = c^2$

And then we write down what we need to find.

Change in a with respect to time when $b = 1/2$ $\quad \dfrac{da}{dt}$

Since we know we need to solve for da/dt, we can expect to use implicit differentiation to take the derivative of the equation of the right triangle, since taking the derivative of that equation with respect to time will give us da/dt on the left.

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So, starting with the equation of the triangle, we'll first plug in any values we already have, in this case $c = 1$.

$$a^2 + b^2 = c^2$$

$$a^2 + b^2 = (1)^2$$

$$a^2 + b^2 = 1$$

Now we'll use implicit differentiation to take the derivative of both sides of the equation with respect to time.

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$$

We're looking for the change in a with respect to time, so we need to solve for da/dt.

$$2a\frac{da}{dt} = -2b\frac{db}{dt}$$

$$\frac{da}{dt} = \frac{-2b\frac{db}{dt}}{2a}$$

We know that we're trying to find da/dt when $b = 1/2$, and we know that $db/dt = -1/4$. Plugging these into our equation, we get

$$\frac{da}{dt} = \frac{-2\left(\frac{1}{2}\right)\left(-\frac{1}{4}\right)}{2a}$$

$$\frac{da}{dt} = \frac{\frac{1}{4}}{2a}$$

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$$\frac{da}{dt} = \frac{1}{8a}$$

Since we have a leftover a in our answer, it means we need to find the value of a when $b = 1/2$.

$$a^2 + b^2 = 1$$

$$a^2 + \left(\frac{1}{2}\right)^2 = 1$$

$$a^2 = \frac{3}{4}$$

$$a = \pm\sqrt{\frac{3}{4}}$$

We can discount the negative value because the distance along the ground between the fence and the bottom of the shovel can't be negative (we're talking about distances in real space!). Therefore

$$a = \sqrt{\frac{3}{4}}$$

Plugging this into our equation for da/dt gives

$$\frac{da}{dt} = \frac{1}{8a}$$

$$\frac{da}{dt} = \frac{1}{8\sqrt{\frac{3}{4}}}$$

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$$\frac{da}{dt} = \frac{1}{\frac{8}{2}\sqrt{3}}$$

$$\frac{da}{dt} = \frac{1}{4\sqrt{3}}$$

The fact that da/dt is positive tells us that the distance along the ground between the fence and the bottom of the shovel is increasing, and we can say that the shovel is sliding along the ground at a rate of

$$\frac{1}{4\sqrt{3}} \text{ m/s}$$

which, when we rationalize the denominator is

$$\frac{1}{4\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) \text{ m/s}$$

$$\frac{\sqrt{3}}{12} \text{ m/s}$$

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Topic: Ladder sliding down the wall

Question: A 5 foot ladder is sliding down a vertical wall while its bottom slides away from the wall at 3 ft/s. How fast is the top moving when the top is 4 feet off the ground?

Answer choices:

A $-\frac{9}{4}$ ft/s

B $-\frac{4}{9}$ ft/s

C $-\frac{3}{2}$ ft/s

D $-\frac{2}{3}$ ft/s

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Solution: A

To solve for the rate of change of a ladder sliding down a wall over time, we'll need to use the Pythagorean theorem

$$a^2 + b^2 = c^2$$

where a is the distance from the top of the ladder to the ground on the wall, b is the distance from the wall to the bottom of the ladder on the ground, and c is the length of the ladder.

First you'll need to take the derivative of the formula for a triangle with respect to time. Since c is a constant (the length of the ladder does not change) it will become 0 when we differentiate.

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$$

We already know from the question that

$$\frac{db}{dt} = 3$$

$$a = 4$$

$$c = 5$$

First we need to find the distance for b. We can do this using $a^2 + b^2 = c^2$, $a = 4$ and $c = 5$.

$$a^2 + b^2 = c^2$$

$$(4)^2 + b^2 = (5)^2$$

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$$16 + b^2 = 25$$

$$b^2 = 9$$

$$b = 3$$

Finally, knowing that $db/dt = 3$, $a = 4$ and $b = 3$, we can plug everything into the formula.

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$$

$$2(4)\frac{da}{dt} + 2(3)(3) = 0$$

$$8\frac{da}{dt} + 18 = 0$$

$$8\frac{da}{dt} = -18$$

$$\frac{da}{dt} = -\frac{9}{4}$$

The fact that this is negative indicates that the ladder is sliding down the wall, not up the wall. The rate at which the ladder is sliding down the wall is $-9/4$ ft/s.

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Topic: Ladder sliding down the wall

Question: A 13 foot ladder is sliding down a vertical wall while its bottom slides away from the wall at 9 ft/s. How fast is the top moving when the top is 5 feet off the ground?

Answer choices:

A -2.16 ft/s

B -21.6 ft/s

C -216 ft/s

D $-2{,}160$ ft/s

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Solution: B

To solve for the rate of change of a ladder sliding down a wall over time, we'll need to use the Pythagorean theorem

$$a^2 + b^2 = c^2$$

where a is the distance from the top of the ladder to the ground on the wall, b is the distance from the wall to the bottom of the ladder on the ground, and c is the length of the ladder.

First you'll need to take the derivative of the formula for a triangle with respect to time. Since c is a constant (the length of the ladder does not change) it will become 0 when we differentiate.

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$$

We already know from the question that

$$\frac{db}{dt} = 9$$

$$a = 5$$

$$c = 13$$

First we need to find the distance for b. We can do this using $a^2 + b^2 = c^2$, $a = 5$ and $c = 13$.

$$a^2 + b^2 = c^2$$

$$(5)^2 + b^2 = (13)^2$$

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$$25 + b^2 = 169$$

$$b^2 = 144$$

$$b = 12$$

Finally, knowing that $db/dt = 9$, $a = 5$ and $b = 12$, we can plug everything into the formula.

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 0$$

$$2(5)\frac{da}{dt} + 2(12)(9) = 0$$

$$10\frac{da}{dt} + 216 = 0$$

$$10\frac{da}{dt} = -216$$

$$\frac{da}{dt} = -21.6$$

The fact that this is negative indicates that the ladder is sliding down the wall, not up the wall. The rate at which the ladder is sliding down the wall is -21.6 ft/s.

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